MODULE 4

The mechanical properties, among all the properties of plastic materials, are often the most important properties because virtually all service conditions and the majority of end-use applications involve some degree of mechanical loading. Nevertheless, these properties are the least understood by most design engineers. The material selection for a variety of applications is quite often based on mechanical properties such as tensile strength, modulus, elongation, and impact strength.
The basic understanding of stress–strain behavior of plastic materials is of utmost importance to design engineers. One such typical stress–strain (load-deformation) diagram is illustrated in Figure 2-1. For a better understanding of the stress–strain curve, it is necessary to define a few basic terms that are associated with the stress–strain diagram.

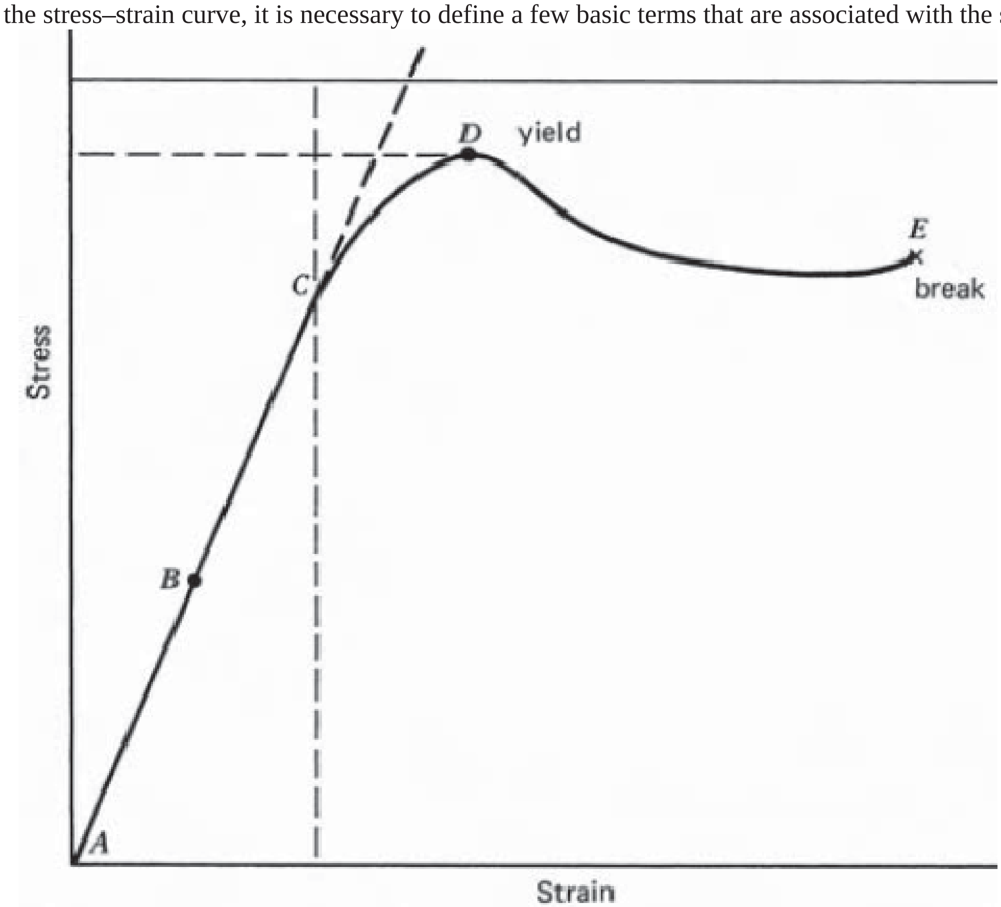


Figure 2-1. A typical stress–strain curve.

Stress. The force applied to produce deformation in a unit area of a test specimen. Stress is a ratio of applied load to the original cross-sectional area expressed in lb/in.2.
Strain. The ratio of the elongation to the gauge length of the test specimen, or simply stated, change in length per unit of the original length (⊗*l*/*l*). It is expressed as a dimensionless ratio.
Elongation. The increase in the length of a test specimen produced by a tensile load.
Yield Point. The fi rst point on the stress–strain curve at which an increase in strain occurs without the increase in stress.
Yield Strength. The stress at which a material exhibits a specifi ed limiting deviation from the proportionality of stress to strain. Unless otherwise specified, this stress will be at the yield point.
Proportional Limit. The greatest stress at which a material is capable of sustaining the applied load without any deviation from proportionality of stress to strain (Hooke's Law). This is expressed in lb/in.2.

Modulus of Elasticity. The ratio of stress to corresponding strain below the proportional limit of a material. It is expressed in *F*/*A*, usually lb/in.^{2} This is also known as Young's modulus. A modulus is a measure of material's stiffness.
Ultimate Strength. The maximum unit stress a material will withstand when subjected to an applied load in compression, tension, or shear. This is expressed in lb/in.^{2}.
Secant Modulus. The ratio of the total stress to corresponding strain at any specific point on the stress–strain curve. It is also expressed in *F*/*A* or lb/in.^{2}.

The stress–strain diagram illustrated in Figure 2-1 is typical of that obtained in tension for a constant rate of loading. However, the curves obtained from other loading conditions, such as compression or shear, are quite similar in appearance. The initial portion of the stress–strain curve between points *A* and *C* is linear and it follows Hooke's law, which states that for an elastic material the stress is proportional to the strain. The point *C* at which the actual curve deviates from the straight line is called the proportional limit, meaning that only up to this point is stress proportional to strain. The behavior of plastic material below the proportional limit is elastic in nature and therefore the deformations are recoverable. The deformations up to point *B* in Figure 2-1 are relatively small and have been associated with the bending and stretching of the interatomic bonds between atoms of plastic molecules as shown in Figure 2-2*a*. This type of deformation is instantaneous and recoverable. There is no permanent displacement of the molecules relative to each other. The deformation that occurs beyond point *C* in Figure 2-1 is similar to a straightening out of a coiled portion of the molecular chains (Figure2-2*b*). There is no intermolecular slippage and the deformations may be recoverable ultimately, but not instantaneously. The extensions that occur beyond the yield point or the elastic limit of the material are not recoverable (Figure 2-2*c*). There deformations occur because of the actual displacement of the molecules with respect to each other. The displaced molecules cannot slip back to their original positions and, therefore, a permanent deformation or set occurs. These three types of deformations, as shown in Figure 2-2, do not occur separately but are superimposed on each other. The bonding and the stretching of the interatomic bonds are almost instantaneous. However, the molecular uncoiling is relatively slow. Molecular slippage effects are the slowest of all three deformations

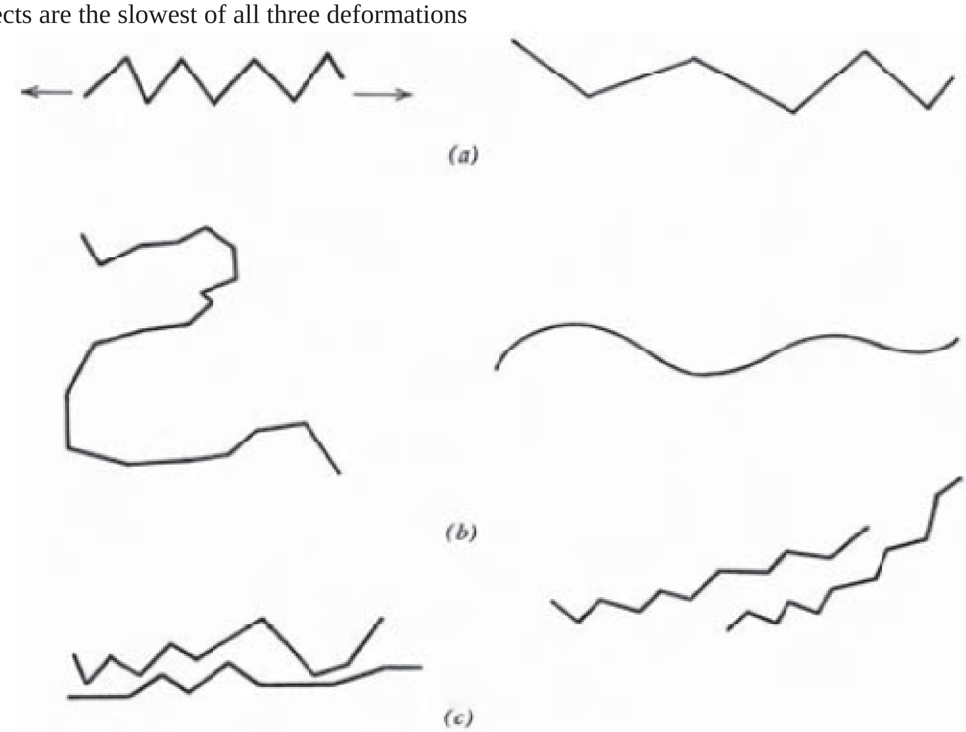
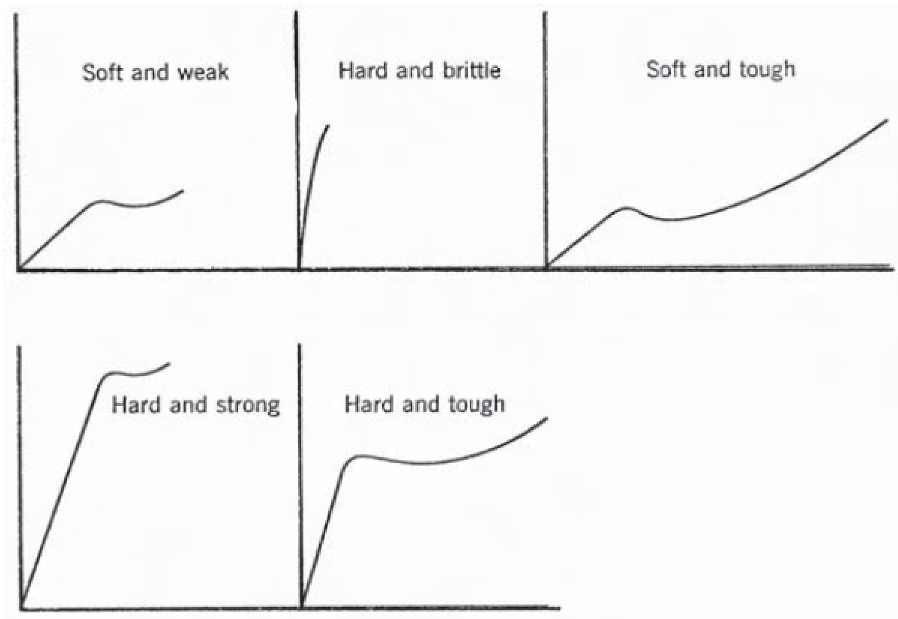


Figure 2-2. Extension types: (*a*) Bond bending, (*b*) uncoiling, (*c*) slippage.

The polymeric materials can be broadly classifi ed in terms of their relative softness, brittleness, hardness, and toughness. The tensile stress–strain diagrams serve as a basis for such a classifi cation (6). The area under the stress–strain

curve is considered as the toughness of the polymeric material. Figure 2-4*a* illustrates typical tensile stress–strain curves for several types of polymeric
materials.
A soft and weak material is characterized by low modulus, low yield stress, and a moderate elongation at break point. Polytetrafl uoroethylene (PTFE) is a good example of one such type of plastic material.
A soft but tough material shows low modulus and low yield stress, but very high elongation and high stress at break. Polyethylene is a classic example of these types of plastics.
A hard and brittle material is characterized by high modulus and low elongation. It may or may not yield before breaking. One such type of polymer is general purpose phenolic.
A hard and strong material has high modulus, high yield stress, usually high ultimate strength, and low elongation. Acetal is a good example of this class of materials.
A hard and tough material is characterized by high modulus, high yield stress, high elongation at break, and high ultimate strength. Polycarbonate is considered a hard and tough material. Figure 2-4*b* illustrates the relation between ductility and strength.



(*a*)

TENSILE TESTS (ASTM D 638, ISO 527-1)
Tensile elongation and tensile modulus measurements are among the most important indications of strength in a material and are the most widely specified properties of plastic materials. Tensile test, in a broad sense, is a measurement of the ability of a material to withstand forces that tend to pull it apart and to determine to what extent the material stretches before breaking. Tensile modulus, an indication of the relative stiffness of a material, can be determined from a stress–strain diagram. Different types of plastic materials are often compared on the basis of tensile strength, elongation, and tensile modulus data. Many plastics are very sensitive to the rate of straining and environmental conditions.
Apparatus
The tensile testing machine of a constant-rate-of-crosshead movement is used. It has a fi xed or essentially stationary member carrying one grip, and a movable member carrying a second grip. Self-aligning grips employed for holding the test specimen between the fixed member and the movable member prevent alignment problems. A controlled-velocity drive mechanism is used. Some of the commercially available machines use a closed-loop servo-controlled drive mechanism to provide a high degree of speed accuracy.
Test Specimens and Conditioning
Test specimens for tensile tests are prepared many different ways. Most often, they are either injection molded or compression molded. The specimens may also be prepared by machining operations from materials in sheet, plate, slab, or similar form. Test specimen dimensions vary considerably depending upon the requirements and are

described in detail in the ASTM book of standards. Figure 2-9 shows ASTM D 638 Type I tensile test specimen most commonly used for testing rigid and semirigid plastics.

The specimens are conditioned using standard conditioning procedures. Since the tensile properties of some plastics change rapidly with small changes in temperature, it is recommended that tests be conducted in the standard laboratory atmosphere of 23 □} 2°C and 50 □} 5 percent relative humidity.

Test Procedures

A. Tensile Strength

The speed of testing is the relative rate of motion of the grips or test fixtures during the test. There are basically five different testing speeds specified in the ASTM D638 Standard. The most frequently employed speed of testing is 0.2 in./min. Whenever possible, the speed indicated by the specification for the material being tested should be used. If a test speed is not given, appropriate speed that causes rupture between 30 sec and 5 min should be chosen. The test specimen is positioned vertically in the grips of the testing machine. The grips are tightened evenly and firmly to prevent any slippage. The speed of testing is set at the proper rate and the machine is started. As the specimen elongates, the resistance of the specimen increases and is detected by a load cell. This load value (force) is recorded by the instrument. Some machines also record the maximum (peak) load obtained by the specimen, which can be recalled after the completion of the test. The elongation of the specimen is continued until a rupture of the specimen is observed. Load value at break is also recorded. The tensile strength at yield and at break (ultimate tensile strength) are calculated.

B. Tensile Modulus and Elongation

Tensile modulus and elongation values are derived from a stress–strain curve. An extensometer is attached to the test specimen as shown in Figure 2-10*a*. The extensometer is a strain gauge type of device that magnifi es the actual stretch of the specimen considerably. Reliability of the strain measurement is affected by the traditional contact extensometers due to the actual physical contact with the test specimen. In recent years, many test equipment manufacturers have developed noncontact measurement systems based on optical, video and laser devices to overcome problems associated with contact extensometers.

FLEXURAL PROPERTIES (ASTM D 790, ISO 178)

The stress–strain behavior of polymers in fl exure is of interest to a designer as well as a polymer manufacturer. Flexural strength is the ability of the material to withstand bending forces applied perpendicular to its longitudinal axis. The stresses induced by the fl exural load are a combination of compressive and tensile stresses. This effect is illustrated in Figure 2-16. Flexural properties are reported are reported and calculated in terms of the maximum stress and strain that occur at the outside surface of the test bar. Many polymers do not break under fl exure even after a large defl ection that makes determination of the ultimate fl exural strength impractical for many polymers. In such cases, the common practice is to report fl exural yield strength when the maximum strain in the outer fi ber of the specimen has reached 5 percent. For polymeric materials that break easily under fl exural load, the specimen is defl ected until a rupture occurs in the outer fi bers.

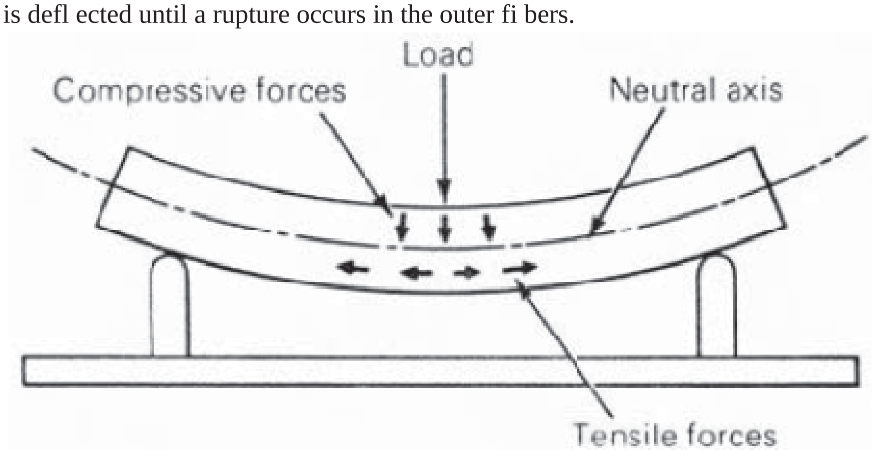


Figure 2-16. Forces involved in bending a simple beam.

There are several advantages of fl exural strength tests over tensile tests (14). If a material is used in the form of a beam and if the service failure occurs in bending, then a fl exural test is more relevant for design or specifi cation purposes than a tensile test, which may give a strength value very different from the calculated strength of the outer fi ber in the bent beam. The fl exural specimen is comparatively easy to prepare without residual strain. The specimen alignment is also more difficult in tensile tests. Also, the tight clamping of the test specimens creates stress concentration points. One other advantage of the fl exural test is that at small strains, the actual deformations are suffi ciently large to be measured accurately.

There are two basic methods that cover the determination of flexural properties of plastics. Method 1 is a three-point loading system utilizing center loading on a simple supported beam. A bar of rectangular cross section rests on two supports and is loaded by means of a loading nose midway between the supports. The maximum axial fiber stresses occur on a line under the loading nose. A closeup of a specimen in the testing apparatus is shown in Figure 2-17. This method is especially useful in determining flexural properties for quality control and specification purposes.

Method 2 is a four-point loading system utilizing two load points equally spaced from their adjacent support points, with a distance between load points of one-third of the support span. In this method, the test bar rests on two supports and is loaded at two points (by means of two loading noses), each an equal distance from the adjacent support point. This arrangement is shown schematically in Figure 2-18.

Method 2 is very useful in testing material that do not fail at the point of maximum stress under a three-point loading system. The maximum axial fiber stress occurs over the area between the loading noses.

Apparatus

Quite often, the machine used for tensile testing is also used for flexural testing. The upper or lower portion of the movable crosshead can be used for flexural testing. The loading nose and support must have cylindrical surfaces. The radius of the nose and the nose support should be at least 1/8 in. to avoid excessive indentation or failure due to stress concentration directly under the loading nose. A strain gauge type of mechanism called a deflectometer or compressometer is used to measure defl ection in the specimen.

Test Specimens and Conditioning

The specimens used for fl exural testing are bars of rectangular cross section and are cut from sheets, plates, or molded shapes. The common practice is to mold the specimens to the desired fi nished dimensions. The specimens are conditioned in accordance with Procedure A of ASTM methods D618 as explained in Chapter 11 of this book. The specimens of size 1/8 · 1/2 · 4 in. are the most commonly used.

Test Procedures and Calculations

The test is initiated by applying the load to the specimen at the specifi ed crosshead rate. The defl ection is measured either by a gauge under the specimen in contact with it in the center of the support span or by measurement of the motion of the loading nose relative to the supports. A load–defl ection curve is plotted if the determination of fl exural modulus value is desired.

The maximum fi ber stress is related to the load and sample dimensions and is calculated using the following equation:

$$\text{Method 1} \quad S = \frac{3PL}{2bd^2}$$

where S = stress (psi); P = load (lb); L = length of span (in.); b = width of specimen (in.); d = thickness of specimen (in.).

Flexural strength is equal to the maximum stress in the outer fibers at the moment of break. This value can be calculated by using the above stress equation by letting load value P equal the load at the moment of break.

For materials that do not break at outer fiber strains up to 5 percent, the flexural yield strength is calculated using the same equation. The load value P in this case is the maximum load at which there is no longer an increase in load with an increase in deflection.

Modulus of Elasticity (Flexural Modulus)

The fl exural modulus is a measure of the stiffness during the fi rst or initial part of the bending process. The fl exural modulus is represented by the slope of the initial straight-line portion of the stress–strain curve and is calculated by dividing the change in stress by the corresponding change in strain. The procedure to calculate flexural modulus is similar to the one described previously for tensile modulus calculations.

they do aid in the understanding and analysis of the behaviour of viscoelastic materials. Some of the more important models will now be considered.

(a) Maxwell Model

The Maxwell Model consists of a spring and dashpot in series at shown in Fig. 2.34. This model may be analysed as follows.

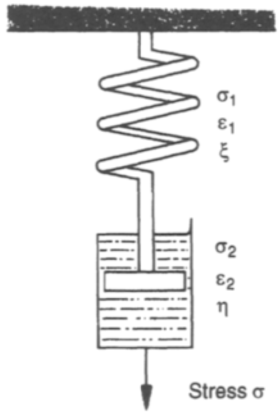


Fig. 2.34 The Maxwell model

Stress–Strain Relations

The spring is the elastic component of the response and obeys the relation

$$\sigma_1 = \xi \cdot \varepsilon_1 \tag{2.27}$$

where σ_1 and ε_1 are the stress and strain respectively and ξ is a constant.

The dashpot is the viscous component of the response and in this case the stress σ_2 is proportional to the rate of strain $\dot{\varepsilon}_2$, ie

$$\sigma_2 = \eta \cdot \dot{\varepsilon}_2 \tag{2.28}$$

where η is a material constant.

Equilibrium Equation

For equilibrium of forces, assuming constant area

$$\text{Applied Stress, } \sigma = \sigma_1 = \sigma_2 \tag{2.29}$$

Geometry of Deformation Equation

The total strain, ε is equal to the sum of the strains in the two elements. So

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \tag{2.30}$$

From equations (2.27), (2.28) and (2.30)

$$\dot{\varepsilon} = \frac{1}{\xi}\dot{\sigma}_1 + \frac{1}{\eta}\sigma_2$$

$$\dot{\varepsilon} = \frac{1}{\xi} \cdot \dot{\sigma} + \frac{1}{\eta} \cdot \sigma \tag{2.31}$$

This is the governing equation of the Maxwell Model. It is interesting to consider the response that this model predicts under three common time-dependent modes of deformation.

(i) Creep

If a constant stress, σ_o, is applied then equation (2.31) becomes

$$\dot{\varepsilon} = \frac{1}{\eta} \cdot \sigma_o \tag{2.32}$$

which indicates a constant rate of increase of strain with time.

From Fig. 2.35 it may be seen that for the Maxwell model, the strain at any time, t, after the application of a constant stress, σ_o, is given by

$$\varepsilon(t) = \frac{\sigma_o}{\xi} + \frac{\sigma_o}{\eta}t$$

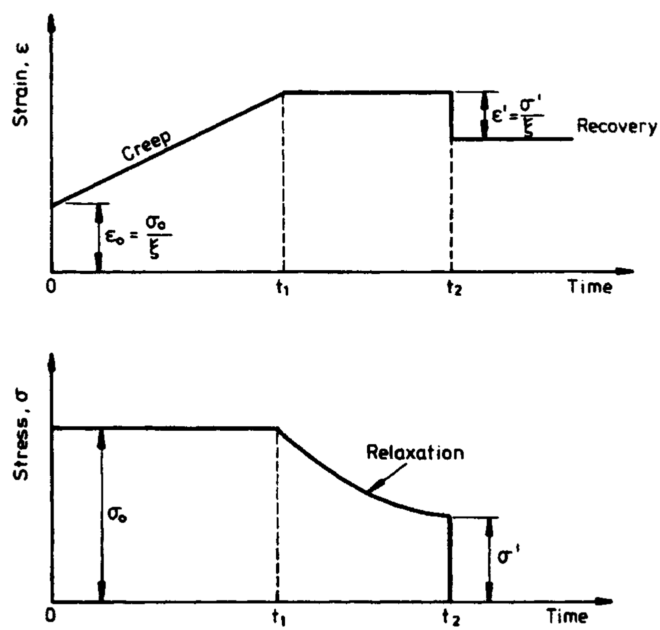


Fig. 2.35 Response of Maxwell model

Hence, the creep modulus, $E(t)$, is given by

$$E(t) = \frac{\sigma_o}{\varepsilon(t)} = \frac{\xi\eta}{\eta + \xi t} \tag{2.33}$$

(ii) Relaxation

If the strain is held constant then equation (2.31) becomes

$$0 = \frac{1}{\xi} \cdot \dot{\sigma} + \frac{1}{\eta} \cdot \sigma$$

Solving this differential equation (see Appendix B) with the initial condition $\sigma = \sigma_o$ at $t = t_o$ then,

$$\sigma(t) = \sigma_o e^{-\frac{\xi}{\eta} t} \tag{2.34}$$

$$\sigma(t) = \sigma_o e^{-t/T_R} \tag{2.35}$$

where $T_R = \eta/\xi$ is referred to as the *relaxation time.*

This indicates that the stress decays exponentially with a time constant of η/ξ (see Fig. 2.35).

(iii) Recovery

When the stress is removed there is an instantaneous recovery of the elastic strain, ε^1, and then, as shown by equation (2.31), the strain rate is zero so that there is no further recovery (see Fig. 2.35).

It can be seen therefore that although the relaxation behaviour of this model is acceptable as a first approximation to the actual materials response, it is inadequate in its prediction for creep and recovery behaviour.

(b) Kelvin or Voigt Model

In this model the spring and dashpot elements are connected in parallel as shown in Fig. 2.36.

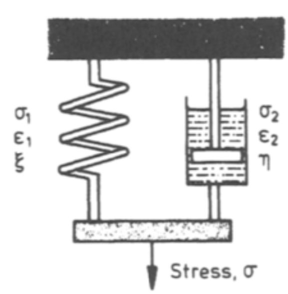


Fig. 2.36 The Kelvin or Voigt Model

Stress–Strain Relations

These are the same as the Maxwell Model and are given by equations (2.27) and (2.28).

Equilibrium Equation

For equilibrium of forces it can be seen that the applied load is supported jointly by the spring and the dashpot, so

$$\sigma = \sigma_1 + \sigma_2 \tag{2.36}$$

Geometry of Deformation Equation

In this case the total strain is equal to the strain in each of the elements, i.e.

$$\varepsilon = \varepsilon_1 = \varepsilon_2 \tag{2.37}$$

From equations (2.27), (2.28) and (2.36)

$$\sigma = \xi \cdot \varepsilon_1 + \eta \dot{\varepsilon}_2$$

or using equation (2.37)

$$\sigma = \xi \cdot \varepsilon + \eta \cdot \dot{\varepsilon} \tag{2.38}$$

This is the governing equation for the Kelvin (or Voigt) Model and it is interesting to consider its predictions for the common time dependent deformations.

(i) Creep

If a constant stress, σ_o, is applied then equation (2.38) becomes

$$\sigma_o = \xi \cdot \varepsilon + \eta \dot{\varepsilon}$$

and this differential equation may be solved for the total strain, ε, to give

$$\varepsilon(t) = \frac{\sigma_o}{\xi}\left[1 - e^{-\frac{\xi}{\eta}t}\right]$$

where the ratio η/ξ is referred to as the *retardation time*, T_R.

This indicates an exponential increase in strain from zero up to the value, σ_o/ξ, that the spring would have reached if the dashpot had not been present. This is shown in Fig. 2.37. As for the Maxwell Model, the creep modulus may be determined as

$$E(t) = \frac{\sigma_o}{\varepsilon(t)} = \xi\left[1 - e^{-\frac{t}{T_R}}\right]^{-1} \tag{2.39}$$

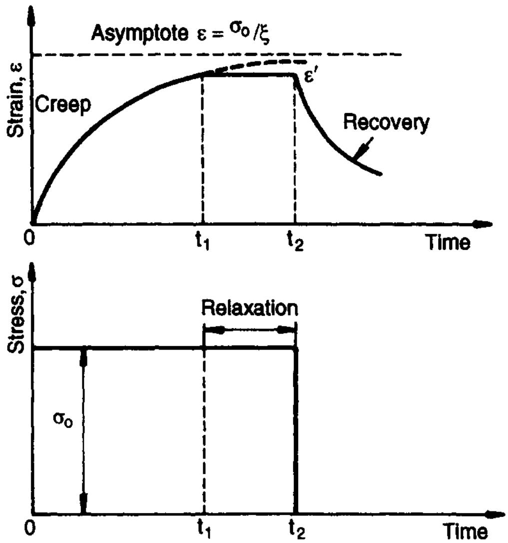


Fig. 2.37 Response of Kelvin/Voigt model

(ii) Relaxation

If the strain is held constant then equation (2.38) becomes

$$\sigma = \xi \cdot \varepsilon$$

That is, the stress is constant and supported by the spring element so that the predicted response is that of an elastic material, i.e. no relaxation (see Fig. 2.37)

(iii) Recovery

If the stress is removed, then equation (2.38) becomes

$$0 = \xi \cdot \varepsilon + \eta\dot{\varepsilon}$$

Solving this differential equation with the initial condition $\varepsilon = \varepsilon'$ at the time of stress removal, then

$$\varepsilon(t) = \varepsilon' e^{-\frac{\xi t}{\eta}} \tag{2.40}$$

This represents an exponential recovery of strain which is a reversal of the predicted creep.

(c) More Complex Models

It may be seen that the simple Kelvin model gives an acceptable first approximation to creep and recovery behaviour but does not account for relaxation. The Maxwell model can account for relaxation but was poor in relation to creep and recovery. It is clear therefore that some compromise may be achieved by combining the two models. Such a set-up is shown in Fig. 2.38. In this case the stress–strain relations are again given by equations (2.27) and (2.28). The geometry of deformation yields.

$$\text{Total strain, } \varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_k \quad (2.41)$$

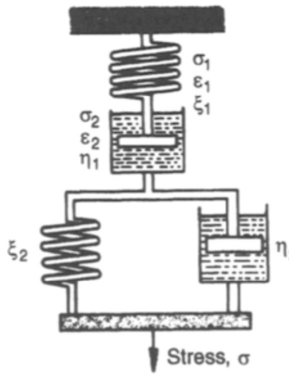


Fig. 2.38 Maxwell and Kelvin models in series

where ε_k is the strain response of the Kelvin Model. From equations (2.27), (2.28) and (2.41).

$$\varepsilon(t) = \frac{\sigma_o}{\xi_1} + \frac{\sigma_o t}{\eta_1} + \frac{\sigma_o}{\xi_2}\left[1 - e^{-\frac{\xi_2 t}{\eta_2}}\right] \quad (2.42)$$

From this the strain rate may be obtained as

$$\dot{\varepsilon} = \frac{\sigma_o}{\eta_1} + \frac{\sigma_o}{\eta_2} e^{-\frac{\xi_2}{\eta_2} t} \quad (2.43)$$

The response of this model to creep, relaxation and recovery situations is the sum of the effects described for the previous two models and is illustrated in Fig. 2.39. It can be seen that although the exponential responses predicted in these models are not a true representation of the complex viscoelastic response of polymeric materials, the overall picture is, for many purposes, an acceptable approximation to the actual behaviour. As more and more elements are added to the model then the simulation becomes better but the mathematics become complex.

Example 2.12 An acrylic moulding material is to have its creep behaviour simulated by a four element model of the type shown in Fig. 2.38. If the creep curve for the acrylic at 14 MN/m^2 is as shown in Fig. 2.40, determine the values of the four constants in the model.

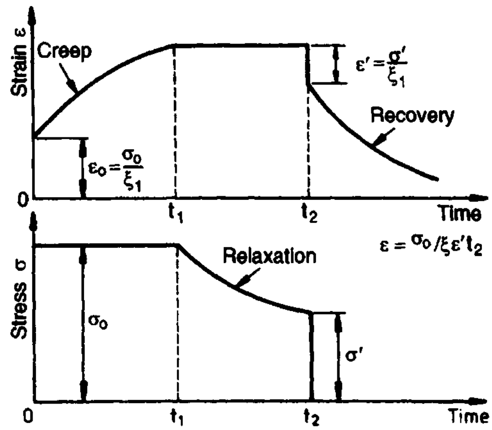


Fig. 2.39 Response of combined Maxwell and Kelvin models

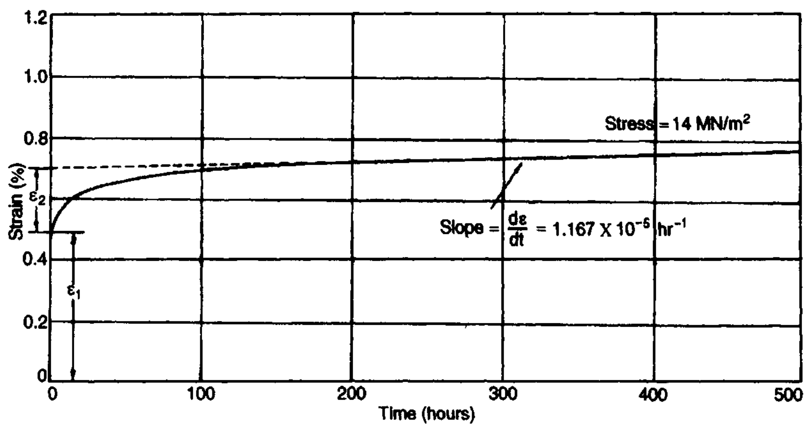


Fig. 2.40 Creep curve for acrylic at 20°C

Solution The spring element constant, ξ_1, for the Maxwell model may be obtained from the instantaneous strain, ε_1. Thus

$$\xi_1 = \frac{\sigma_o}{\varepsilon_1} = \frac{14}{0.005} = 2800 \text{ MN/m}^2$$

The dashpot constant, η_1, for the Maxwell element is obtained from the slope of the creep curve in the steady state region (see equation (2.32)).

$$\eta_1 = \frac{\sigma_o}{\dot{\varepsilon}} = \frac{14}{1.167 \times 10^{-6}} = 1.2 \times 10^7 \text{ MN.hr/m}^2$$

$$= 4.32 \times 10^{10} \text{ MN.s/m}^2$$

The spring constant, ξ_2, for the Kelvin–Voigt element is obtained from the maximum retarded strain, ε_2, in Fig. 2.40.

$$\xi_2 = \frac{\sigma_o}{\varepsilon_2} = \frac{14}{(0.7 - 0.5)10^{-2}} = 7000 \text{ MN/m}^2$$

The dashpot constant, η_2, for the Kelvin–Voigt element may be determined by selecting a time and corresponding strain from the creep curve in a region where the retarded elasticity dominates (i.e. the knee of the curve in Fig. 2.40) and substituting into equation (2.42). If this is done then $\eta_2 = 3.7 \times 10^8$ MN.s/m^2.

Having thus determined the constants for the model the strain may be predicted for any selected time or stress level assuming of course these are within the region where the model is applicable.

(d) Standard Linear Solid

Another model consisting of elements in series and parallel is that attributed to Zener. It is known as the Standard Linear Solid and is illustrated in Fig. 2.41. The governing equation may be derived as follows.

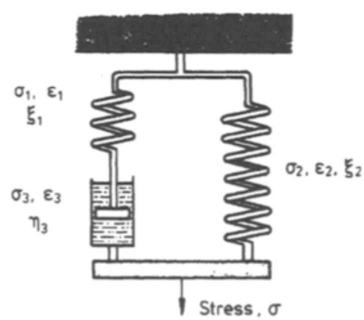


Fig. 2.41 The standard linear solid

Stress–Strain Relations

As shown earlier the stress–strain relations are

$$\sigma_1 = \xi_1 \varepsilon_1 \tag{2.44}$$

$$\sigma_2 = \xi_2 \varepsilon_2 \tag{2.45}$$

$$\sigma_3 = \eta_3 \dot{\varepsilon}_3 \tag{2.46}$$

Equilibrium Equation

In a similar manner to the previous models, equilibrium of forces yields.

$$\sigma_1 = \sigma_3$$

$$\sigma = \sigma_1 + \sigma_2 \tag{2.47}$$